**12-9**

1. 6.396
2. (0,0,1,0,0), 12
3. Lower bound is 6.396 and upper bound is 12, which comes from part (a) and (b), respectively.
4. (1,1,0,0,0), objective value=9



The optimal value is between the lower and upper bound defined in (c)

**12-14**

1. Valid, it strengthen the original LP relaxation.

Objective value: -42 🡪 -29.333 (x=(2/3,1,1,0) y=2/3)

1. Vaild, same constraint.

Objective value: -42 🡪 -42 (x=(1,1,0,0), y=1/2)

1. Invalid, from original constraints, x=(1,1,1,1), y=1 is feasible. However, with additional constraint, it is infeasible.
2. Vaild, it strengthen the original LP relaxation.

Objective value: -42 🡪 -39.333 (x=(1,1,2/3,0), y=2/3)

**12-21**

[1,0,0,0] : infeasible

[0,1,0,0] : infeasible

[0,0,1,0] : infeasible

[0,0,0,1] : infeasible

[1,1,0,0] : infeasible

[1,0,1,0] : 45

[1,0,0,1] : 35

[1,1,1,0] : infeasible

[1,1,0,1] : infeasible

[1,0,1,1] : infeasible

[1,1,1,1] : infeasible

[0,1,0,1] : infeasible

[0,1,1,0] : 40

[0,0,1,1] : infeasible

[0,1,1,1] : infeasible

[0,0,0,0] : infeasible

s.t.

1. The dualization partially relax some of the main, linear constraints of an ILP by moving them to the objective function. The optimal value of any Lagrangian relaxation of a maximization problem using valid multipliers yields an upper bound on the optimal values of the full model.

[1,0,0,0] : 15

[0,1,0,0] : 10

[0,0,1,0] : 30

[0,0,0,1] : 20

[1,1,0,0] : 25

[1,0,1,0] : 45

[1,0,0,1] : 35

[1,1,1,0] : 55

[1,0,1,1] : 65

[1,1,0,1] : infeasible

[1,1,1,1] : infeasible

[0,0,1,1] : 50

[0,1,0,1] : infeasible

[0,1,1,0] : 40

[0,1,1,1] : infeasible

[0,0,0,0] : 0

The relaxation optimal value: 0 🡪 < 35(optimal value) 🡪 lower bound



[1,0,0,0] : 27

[0,1,0,0] : 22

[0,0,1,0] : 40

[0,0,0,1] : 30

[1,1,0,0] : 27

[1,0,1,0] : 45

[1,0,0,1] : 35

[1,0,1,1] : 53

[1,1,0,1] : infeasible

[1,1,1,0] : 45

[1,1,1,1] : infeasible

[0,0,1,1] : 42

[0,1,0,1] : infeasible

[0,1,1,0] : 40

[0,1,1,1] : infeasible

[0,0,0,0] : 22

The relaxation optimal value: 22 🡪 < 35(optimal value) 🡪 lower bound



[1,0,0,0] : 115

[0,1,0,0] : 110

[0,0,1,0] : 230

[0,0,0,1] : 220

[1,1,0,0] : 25

[1,0,1,0] : 45

[1,0,0,1] : 35

[0,0,1,1] : 150

[0,1,0,1] : infeasible

[1,0,1,1] : -35

[1,1,0,1] : infeasible

[1,1,1,0] :-145

[1,1,1,1] : infeasible

[0,1,1,0] : 40

[0,1,1,1] : infeasible

[0,0,0,0] : 300

The relaxation optimal value: -145 🡪 < 35(optimal value) 🡪 lower bound

1. Apply sub-gradient method for Lagrangian dual. Let step size 1 and initial be 50,25. Iterating 100 times, I found that the objective value of Lgrangian subproblem reaches maximum when where the objective value is 35. Please refer to the code for detailed algorithm.

**4.**

[*a,b,c,d*] – *a* means the number of width 45 in the pattern, *b* - 36, *c* – 30, *d* - 15

I generated initial patterns, [2,0,0,0], [0,2,0,0], [0,0,3,0], [0,0,0,6] of length 100.

Then I solved master problem to find price and sub problem to find pattern, iteratively. As a result, I could generate patterns, [0, 2, 0, 1], [1, 1, 0, 1], [0, 1, 2, 0], [0, 1, 0, 4]. There is no beneficial pattern from the stock of length 50. The detailed algorithm is described in the codebase I attached.

Total patterns generated: [2,0,0,0], [0,2,0,0], [0,0,3,0], [0,0,0,6], [0, 2, 0, 1], [1, 1, 0, 1], [0, 1, 2, 0], [0, 1, 0, 4].

Minimize 2000 x\_1 + 2000 x\_2 + 2000 x\_3 + 2000 x\_4 + 2000 x\_5 + 2000 x\_6 + 2000 x\_7 + 2000 x\_8

Subject To

c1: 2 x\_1 + x\_6 >= 100

c2: 2 x\_2 + 2 x\_5 + x\_6 + x\_7 + x\_8 >= 500

c3: 3 x\_3 + 2 x\_7 >= 495

c4: 6 x\_4 + x\_5 + x\_6 + 4 x\_8 >= 250

The optimal solution is [0,0,0,0,66,100,248,21].

The objective value is 870000.